Cooling a mechanical resonator via coupling to a tunable double quantum dot

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We study the cooling of a mechanical resonator (MR) that is capacitively coupled to a double quantum dot (DQD). The MR is cooled by the dynamical backaction induced by the capacitive coupling between the DQD and the MR. The state transition between the two dots of the DQD is excited by an ac field and afterward a tunneling event results in the decay of the excited state of the DQD. An important advantage of this system is that both the energy-level splitting and the decay rate of the DQD can be well tuned by varying the gate voltage. We find that the steady average occupancy, below unity, of the MR can be achieved by changing both the decay rate of the excited state and the red-detuning between the transition frequency of the DQD and the microwave frequency, in analogy to the laser sideband cooling of an atom or trapped ion in atomic physics. Our results show that the cooling of the MR to the ground state is experimentally implementable. Also, the MR can be heated and the steady-state average occupancy becomes infinite when the microwave frequency is larger than the transition frequency of the DQD (i.e., the blue-detuning).

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I. INTRODUCTION

Mechanical resonators (MRs) are currently attracting considerable interest because of their potential applications to high-precision displacement detection,[1] mass detection,[2] and quantum measurement.[3] Recent technical advances allow the fabrication of a MR with both a high quality factor (Q factor) and a sufficient high frequency, approaching 1 GHz.[4–6] Such a MR provides a good platform for exploring various quantum phenomena and for observing the quantum-to-classical transition in macroscopic objects.[7,8] Moreover, quantized MRs could be useful in quantum information science. Indeed, the quantized motion of buckling nanoscale bars has been proposed for implementing qubits.[9–11] The generation of entangled states[12,13] and squeezed states[14] and quantum nondemolition measurements[15] using MRs have also been studied. However, to prepare an ideal ground state, the basic requirement is to be able to cool the MR to a state with a mean phonon number $\langle n\rangle \ll 1$.

Numerous experiments on cooling a single MR have been reported recently (see, e.g., Refs. 16–22). In these experiments, the MR and a fixed micromirror form an optical cavity and the MR is cooled by either radiation-pressure-induced backaction or bolometric backaction. Experimental results show that a single MR can be cooled down from room temperature to an effective temperature on the order of 0.1 K (Refs. 16 and 17) or 10 K.[18–20] However, for a MR with frequency of ~20 MHz, a temperature lower than 1 mK is required in order to drive the MR to the quantum regime. Thus, more effective cooling methods are needed, in addition to increasing the oscillation frequency of the MR. Moreover, besides the classical and semiclassical analyses of cooling a single MR via dynamical backaction,[20,23,24] some quantum theories have also been developed.[25–28] For example, in Refs. 25 and 26, it is predicted that a MR can be cooled down to its ground state when the frequency ω_m of the MR is either comparable to or larger than the optical cavity's resonance linewidth Γ. In laser cooling, this corresponds to the sideband cooling of a bound atom or a trapped ion, where the lowest occupancy attainable is given by $\langle n\rangle \approx \Gamma^2/16\omega_m^2 \ll 1$, indicating that the MR can be most of the time in its ground state. Recently, the resolved-sideband cooling of a MR has been realized by coupling the MR to an optical resonant system.[29]

Besides optomechanical cooling, an alternative way would be to cool the MR by coupling it, via an electronic coupling, to an electronic system. This provides the advantage of fabricating and integrating the electronic device into a cryogenic system. In principle, the electronic cooling of a MR can be achieved by several means, including coupling the MR to: (1) an optical quantum dot (QD),[30] (2) a Josephson-junction superconducting quantum device[31–35] (which behaves like a superconducting artificial atom[36]), and (3) a one-dimensional transmission line.[37] Moreover, the experimental cooling of a MR, via coupling it to a superconductor single-electron transistor[38] or to an LC circuit,[39] has also been reported.

Cooling a mechanical resonator coupled to a double quantum dot

In this work we propose an approach to cool a MR by coupling it to an electronic system: a double quantum dot (DQD). Indeed, the whole system consists of a DQD and a MR. The MR, together with another static plate, forms a gate capacitor adjacent to the left dot (see Fig. 1). The oscillation of the MR will modulate the effective capacitance of this capacitor. In this way, the MR can be strongly coupled to the DQD.

The cooling mechanism can be understood as follows. Two localized states in the DQD ($|1\rangle$ and $|2\rangle$), with energy-level splitting $\hbar\omega_0$, are driven by an ac microwave field of

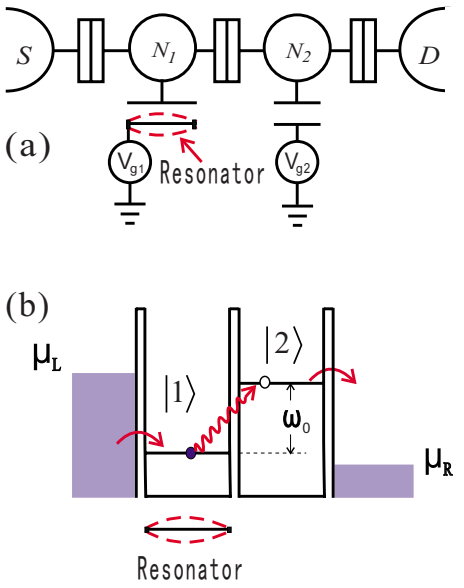

FIG. 1. (Color online) (a) Schematic diagram of a DQD connected to an electron source S and drain D via tunneling barriers. An oscillating plate (the MR) and another static plate form a capacitor on the left dot, which provides a capacitive coupling between the oscillating MR and the DQD. The energy level of each dot is tunable by varying the gate voltage V_g applied to the dot through the capacitor. (b) Transport process of an electron through a DQD: First, an electron tunnels from the source to the left dot, and then a microwave field drives it to the right dot. Finally, it tunnels to the drain on the right side.

frequency ω_d (Fig. 1). This ac field, applied to the tunnel barrier, can excite the DQD from an initial state $|1\rangle$ to the state $|2\rangle$. Similar to the resolved-sideband cooling of a trapped ion, the DQD resonant transition frequency is modulated by the oscillation of the MR, and the absorption spectrum consists of a series of sidebands at frequencies $(\omega_0 - j\omega_m)$, where $j = \pm 1, \pm 2, \ldots$. When the energy-level splitting $\hbar\omega_0$ of the DQD is tuned to satisfy the lowest sideband condition, i.e., $\omega_m = \omega_0 - \omega_d$, the excitation of the DQD from the ground state $|1\rangle$ to the excited state $|2\rangle$ absorbs a photon of energy $\hbar(\omega_0 - \omega_m)$. The subsequent decay of the DQD via electron tunneling emits a photon of energy $\hbar\omega_0$. Hence, each scattering process carries away the MR's vibrational energy by $\hbar\omega_m$, or reduces the MR's quantum number n by 1. This cooling process is described by the state transition $|1\rangle|n\rangle \to |2\rangle|n-1\rangle \to |1\rangle|n-1\rangle$ (see Fig. 2). A series of cycles of this process leads to the cooling of the MR. On the other hand, the reverse process $|1\rangle|n\rangle \to |2\rangle|n+1\rangle \to |1\rangle|n+1\rangle$, increasing the phonon number, is suppressed because it is off-resonance. Like in Sisyphus cooling[40] (cooling wherein the atom spends more of its time climbing than descending the potential hills associated with the ac-Stark-shifted levels, and loses kinetic energy in doing so), a cycle in one direction induces cooling, while the cycle in the other direction produces heating (see Fig. 2).

A similar approach of cooling a semiconductor beam by coupling it to an optical QD was proposed in Ref. 30. Comparing it to this study, our approach has the following potential advantages:

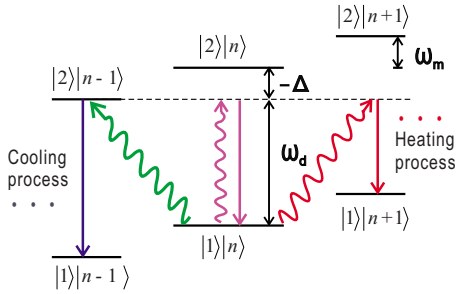

FIG. 2. (Color online) Schematic diagram of the cooling (heating) process in the coupled MR-DQD system. When the DQD is excited by a red-detuned microwave field, i.e., $\Delta = \omega_d - \omega_0 < 0$, the anti-Stokes process $(|1\rangle|n\rangle \to |2\rangle|n-1\rangle)$ is resonantly enhanced. A subsequent decay from the excited state $|2\rangle$ to the ground state $|1\rangle$ reduces the energy of the MR by one quanta. This cools the MR because the emitted (blue) photon has more energy than the absorbed (green) photon. Due to the off-resonance, the Stokes process $(|1\rangle|n\rangle \to |2\rangle|n+1\rangle)$ is suppressed. However, in the resonant case, the Stokes process dominates and the cycle heats the system. The blue (red) vertical downward arrow shows the cooling (heating) process, which decreases (increases) the phonon occupancy n of the MR.

(i) The cooling system can be fabricated more easily. The MR here is used as a part of the capacitor, which is easier to fabricate compared to the proposal in Ref. 30 that embeds an optical QD in a nanoscale beam.

(ii) The decay rate Γ of the upper state of the resonant system (DQD) is just the rate of the electron tunneling to the electrode, which is tunable by varying the gate voltage.

(iii) With a DQD, it is easy to achieve the lowest sideband condition, i.e., $\omega_m = \omega_0 - \omega_d$, for cooling the MR by changing the energy-level splitting $\hbar\omega_0$ via the gate voltage.

In typical transport experiments[41,42] with QDs, the tunneling rate Γ ranges from 10 kHz to 10 GHz, while the fundamental frequency ω_m of the MR is on the order[43] of 100 MHz. The resolved-sideband-cooling regime, i.e., $\omega_m \gg \Gamma$, can be reached by tuning the tunneling rate Γ. In this regime, our results show that the steady average phonon occupancy of the MR can be on the order of $(\Gamma/\omega_m)^2 \ll 1$, indicating that the MR can be cooled to the ground state.

This paper is organized as follows. In Sec. II, we introduce a model of the coupled MR-DQD system and derive its effective Hamiltonian. In Sec. III, we derive the master equation of the coupled MR-DQD system and then eliminate the DQD's degrees of freedom to obtain the master equation for the reduced density matrix of the MR. With this master equation for the MR, we study, in Sec. IV, the cooling of the MR by considering the steady average occupancy of the MR. Moreover, we analyze the steady phonon occupancy of the MR in the resolved-sideband-cooling regime. Section V summarizes our conclusions. Furthermore, in Appendixes A and B, we show the derivations of the master equations.

II. MECHANICAL RESONATOR COUPLED TO A DOUBLE QUANTUM DOT

A. Model

The circuit diagram of a MR coupled to a DQD is shown in Fig. 1(a). The DQD is connected to two electrodes by

tunneling barriers. The bias voltage across the DQD is set such that the chemical potential of the left electrode μ_L is higher than that of the right electrode μ_R, and thus electrons can tunnel from the left electrode to the right one through the DQD. We assume that the DQD is in the Coulomb regime, such that at most a single electron is allowed in the DQD. The corresponding electron states of the DQD are the vacuum state $|0\rangle$, one electron in the left dot $|1\rangle$, and one electron in the right dot $|2\rangle$. Here we consider the case where the hopping strength between the two dots is much smaller than the energy-level splitting of the two dots. To excite the electron from the left dot to the right one, we apply a microwave field to the DQD [see Fig. 1(b)].

As shown in Fig. 1(a), the capacitor adjacent to the left dot is formed by a static plate and a single MR with a gate voltage V_{g1} applied to it. Thus, the displacement of the MR from its equilibrium position will modulate the capacitance $c_{g1}(x)$. For typical experimental parameters, the displacement x of the MR is much less than the equilibrium distance d between the two plates, i.e., $x \ll d$. Hence, the capacitance can be approximately given by

$$c_{g1}(x) \approx C_{g1}\left(1 - \frac{x}{d}\right), \tag{1}$$

where C_{g1} is the capacitance at $x=0$. For the harmonically oscillating MR discussed here, the quantized displacement operator x can be written as (here $\hbar = 1$)

$$x = \sqrt{\frac{1}{2m\omega_m}}(b^\dagger + b), \tag{2}$$

where m is the effective mass of the MR and b^+ (b) is the bosonic creation (annihilation) operator. The total Hamiltonian of the whole system reads

$$H_{\text{tot}} = H_0 + H_{\text{int}} + H_T, \tag{3}$$

where

$$H_0 = H_{\text{leads}} + H_{\text{DQD}} + H_R, \tag{4}$$

is the sum of the isolated bath Hamiltonian H_{leads}, the Hamiltonian H_{DQD} of the DQD driven by a microwave field, and the Hamiltonian H_R of the MR, with

$$H_{\text{leads}} = \sum_{\alpha k} E_{\alpha k} c^\dagger_{\alpha k} c_{\alpha k}, \tag{5}$$

$$H_{\text{DQD}} = \frac{\omega_0}{2}\sigma_z + \Omega\sigma_x + \Omega_0 \cos(\omega_d t)\sigma_x, \tag{6}$$

$$H_R = \omega_m b^\dagger b. \tag{7}$$

Here $c^\dagger_{\alpha k\sigma}$ $(c_{\alpha k\sigma})$ is the creation (annihilation) operator of an electron with momentum k in electrode α $(\alpha = l, r)$. $\sigma_z = a_2^\dagger a_2 - a_1^\dagger a_1$ and $\sigma_x = a_2^\dagger a_1 + a_1^\dagger a_2$ are the Pauli matrices with $a_1^\dagger$ $(a_2^\dagger)$ being the electron creation operator in the left (right) dot of the DQD. The second term in Eq. (6), $\Omega\sigma_x$, is the hopping tunneling term between the two dots. The third term in Eq. (6) describes the applied microwave field with driving frequency ω_d and amplitude Ω_0.[44]

The coupling between the MR and the electron in the left dot is given by[45]

$$H_{\text{int}} = -\lambda a_1^\dagger a_1 (b^\dagger + b), \tag{8}$$

with an electromechanical coupling strength $\lambda = \eta\omega_m$. For a typical electromechanical coupling, $\eta \sim 10^{-1}$. The tunneling coupling between the DQD and the electrodes is

$$H_T = \sum_k (\Omega_{lk} a_1^\dagger c_{lk} + \Omega_{rk} a_2^\dagger c_{rk} + \text{H.c.}), \tag{9}$$

where $\Omega_{lk(rk)}$ characterizes the coupling strength between the QD and the left (right) lead. Hereafter, the subscript "l" ("r") refers to the left (right) electrode.

B. Effective Hamiltonian

It is difficult to directly analyze the coupled system due to the different time scales for the dynamics of the DQD, the MR, and the coupling between them. In order to solve this problem, we first eliminate the coupling term between the MR and the DQD through a canonical transform $U = e^S$ on the whole system, where

$$S = -\eta a_1^\dagger a_1 (b^\dagger - b). \tag{10}$$

With the relations

$$U a_1 U^\dagger = a_1 \exp[\eta(b^\dagger - b)],$$

$$U a_2 U^\dagger = a_2,$$

$$U b U^\dagger = b + \eta a_1^\dagger a_1, \tag{11}$$

the transformed Hamiltonian is given by

$$\begin{aligned} H = U H_{\text{tot}} U^\dagger = & \sum_{\alpha k} E_{\alpha k} c^\dagger_{\alpha k} c_{\alpha k} + \frac{\omega_0}{2}\sigma_z + \omega_m b^\dagger b \\ & + [\Omega + \Omega_0 \cos(\omega_d t)](\sigma_+ B^\dagger + \text{H.c.}) \\ & + \sum_k (\Omega_{lk} a_1^+ c_{lk} B^\dagger + \Omega_{rk} a_2^\dagger c_{rk} + \text{H.c.}), \end{aligned} \tag{12}$$

where we have redefined the energy-level splitting ω_0 and

$$B = \exp[-\eta(b^\dagger - b)]. \tag{13}$$

Here we introduce the ladder operators $\sigma_+ = a_2^\dagger a_1$ and $\sigma_- = \sigma_+^\dagger$.

Moreover, to eliminate the time dependence of the driving term in Eq. (12), we now employ a unitary transform

$$U_R = \exp\left[-i\frac{\omega_d}{2}\sigma_z t\right] \tag{14}$$

to change the Hamiltonian to a rotating frame. By neglecting the fast-oscillating terms within the rotating-wave approximation, the resulting Hamiltonian is given by

$$H = H_{\rm sys} + \sum_{\alpha k} E_{\alpha k} c_{\alpha k}^{\dagger} c_{\alpha k} + \sum_{k} \left[\Omega_{lk} a_1^{+} c_{lk} B^{\dagger} \exp\left(-i\frac{\omega_d}{2}t\right) + \Omega_{rk} a_2^{\dagger} c_{rk} \exp\left(i\frac{\omega_d}{2}t\right) + \text{H.c.} \right], \quad (15)$$

with

$$H_{\rm sys} = -\frac{\Delta}{2}\sigma_z + \omega_m b^{\dagger} b + \frac{\Omega_0}{2}(\sigma_+ B^{\dagger} + \text{H.c.}). \quad (16)$$

Here, $\Delta = \omega_d - \omega_0$ is the driving frequency detuning the microwave field from the transition frequency of the DQD. The rotating-wave approximation is valid when $\omega_d \gg \Omega$, which corresponds to a weak hopping tunneling between the two dots.

III. MASTER EQUATION FOR THE MECHANICAL RESONATOR

Within the Born-Markov approximation, by integrating over the electrode degrees of freedom, we derive a master equation for the coupled MR-DQD system,

$$\frac{d\rho}{dt} = -i[H_{\rm sys}, \rho] + \mathcal{L}_T \rho + \mathcal{L}_D \rho. \quad (17)$$

Here the Liouvillian operator $\mathcal{L}_T$ presents the tunneling events through the DQD in the presence of a single MR. In Eq. (17) and below, the subscript "T" ("D") refers to the tunneling (dissipation). By expanding B in Eq. (13) up to second order in η and assuming that the energy levels with one-phonon-mediated tunneling are within the bias window,[46] $\mathcal{L}_T \rho$ reads

$$\mathcal{L}_T \rho = \Gamma_l (1-\eta^2)\mathcal{D}[a_1^{\dagger}]\rho + \Gamma_r \mathcal{D}[a_2]\rho + \Gamma_l \eta^2 (\mathcal{D}[b a_1^{\dagger}]\rho + \mathcal{D}[b^{\dagger} a_1^{\dagger}]\rho) + \Gamma_l \eta^2 (b^{\dagger} b [a_1, a_1^{\dagger}\rho] + [\rho a_1, a_1^{\dagger}] b^{\dagger} b), \quad (18)$$

with the notation $\mathcal{D}$ for any operator A:

$$\mathcal{D}[A]\rho = A\rho A^{\dagger} - \frac{1}{2}[A^{\dagger} A \rho + \rho A^{\dagger} A]. \quad (19)$$

Here $\Gamma_\alpha = 2\pi \rho_\alpha \Omega_\alpha^2$ is the rate for electron tunneling to the electrode α, while ρ_α denotes the density of states at the electrode α. The Liouvillian operator $\mathcal{L}_D$ describes the intrinsic dissipation of the MR induced by its thermal bath and can be written in a Lindblad form[47] as

$$\mathcal{L}_D \rho = \frac{\gamma}{2}[n(\omega_m)+1][2b\rho b^{\dagger} - (b^{\dagger} b \rho + \rho b^{\dagger} b)] + \frac{\gamma}{2} n(\omega_m)[2b^{\dagger} \rho b - (b b^{\dagger} \rho + \rho b b^{\dagger})], \quad (20)$$

where $\gamma = \omega_m / Q$ is the intrinsic dissipation rate of the MR and $n(\omega_m)$ is the average boson number in the thermal bath.

Next, we focus on the regime in which the driving strength $\eta\Omega_0$ is low enough so that the time scale related to the coupling between the MR and the DQD is slow compared to the dynamical time scale of the DQD and the MR oscillation period, just like the Lamb-Dicke regime considered in laser cooling of an atom or a trapped ion. We also assume that $[n(\omega_m)+1]\gamma$, i.e., the dissipation rate of the MR, is much smaller than the decay rate of the DQD. In this case, the DQD affects the MR perturbatively and can be regarded as an environment (see Ref. 25). Hence, the DQD degrees of freedom can be adiabatically eliminated.[25,48] Since we are interested in the behavior in the limit $t \to \infty$, we can project the system on the subspace with zero eigenvalue of $\mathcal{L}_0$ ($\mathcal{L}_0$ is the Liouvillian for the decoupled MR and DQD), according to

$$\mathcal{P}\rho = \rho_d^s \otimes \text{Tr}_d\{\rho\} = \rho_d^s \otimes \mu, \quad \mathcal{Q} = 1 - \mathcal{P}, \quad (21)$$

with ρ_d^s denoting the stationary (hence the "s" superscript) density matrix of the DQD (hence the "d" subscript), and μ as the density matrix of the MR. Up to second order in η, master equation (17) can be written as[25,48]

$$\frac{d\rho}{dt} = \mathcal{L}(t)\rho = [\mathcal{L}_0(t) + \mathcal{L}_1(t) + \mathcal{L}_2(t)]\rho, \quad (22)$$

where

$$\mathcal{L}_0 \rho = -i[\omega_m b^{\dagger} b, \rho] - i\left[-\Delta\sigma_z + \frac{\Omega_0}{2}\sigma_x, \rho\right] + \Gamma_l \mathcal{D}[a_1^{\dagger}]\rho + \Gamma_r \mathcal{D}[a_2]\rho, \quad (23)$$

$$\mathcal{L}_1 \rho = \mathcal{L}_1^{+}(t)\rho + \mathcal{L}_1^{-}(t)\rho,$$

$$\mathcal{L}_1^{+}\rho = -i\eta\frac{\Omega_0}{2}[(\sigma_+ - \sigma_-)b^{\dagger}, \rho],$$

$$\mathcal{L}_1^{-}\rho = i\eta\frac{\Omega_0}{2}[(\sigma_+ - \sigma_-)b, \rho], \quad (24)$$

$$\mathcal{L}_2 \rho = -\eta^2 \Gamma_l \mathcal{D}[a_1^{\dagger}]\rho + \eta^2 \Gamma_l (\mathcal{D}[b a_1^{\dagger}]\rho + \mathcal{D}[b^{\dagger} a_1^{\dagger}]\rho) + \eta^2 \Gamma_l (b^{\dagger} b [a_1, a_1^{\dagger}\rho] + [\rho a_1, a_1^{\dagger}] b^{\dagger} b) + \mathcal{L}_D \rho - i\eta^2 \frac{\Omega_0}{2}[\sigma_x (b^{\dagger} - b)^2, \rho], \quad (25)$$

are the Liouvillians to zeroth, first, and second orders in η, respectively.

Projecting the master equation in Eq. (22) on the $\mathcal{P}$ subspace, one has

$$\frac{d}{dt}\mathcal{P}\rho = [\mathcal{P}\mathcal{L}_2\mathcal{P} + \mathcal{P}\mathcal{L}_1(-\mathcal{L}_0)^{-1}\mathcal{L}_1\mathcal{P}\rho]. \quad (26)$$

This result is obtained to second-order perturbation in η (see Appendixes A and B). Since we are interested in the dynamics of the MR, we adiabatically eliminate the DQD degrees of freedom.[25,48] From Eq. (26), the master equation for the reduced density matrix of the MR is given by (see Appendix B)

$$\dot{\mu} = -i[\omega_m + \delta_m, b^\dagger b] + \frac{1}{2}\{\gamma[n(\omega_m)+1] + A_-(\omega_m)\}[2b\mu b^\dagger - (b^\dagger b\mu + \mu b^\dagger b)] + \frac{1}{2}[\gamma n(\omega_m) + A_+(\omega_m)][2b^\dagger \mu b - (bb^\dagger \mu + \mu bb^\dagger)], \quad (27)$$

where δ_m is the driving-induced shift of the mechanical frequency.

From Eq. (27), one finds that, besides the effects induced by the coupling to the thermal bath (terms proportional to γ), the cooling and heating induced by the inelastic-scattering processes of the MR can occur and the corresponding rates $A_\mp$ are

$$A_-(\omega) = \frac{\eta^2\Omega_0^2}{2}\mathrm{Re}\left\{\frac{1}{\Delta^2 + (\gamma_0 - i\omega)\left[(\gamma_0 - i\omega) + \frac{\Omega_0^2}{2}L(\omega)\right]} \times \left[\frac{\Omega_0}{2}[P(\omega)\langle\sigma_y\rangle_s + iR(\omega)\langle\sigma_x\rangle_s] + (\gamma_0 - i\omega)\langle\rho_d^1 + \rho_d^2\rangle_s + i\Delta\langle\rho_d^1 - \rho_d^2\rangle_s\right]\right\} + 2D,$$

$$A_+(\omega) = A_-(-\omega), \quad (28)$$

where $\langle\cdots\rangle_s$ means the steady-state solution of the corresponding quantity and

$$L(\omega) = \frac{\Gamma_r + 2\Gamma_l - 2i\omega}{(\Gamma_l - i\omega)(\Gamma_r - i\omega)},$$

$$P(\omega) = \frac{\gamma_0 - i\omega}{\Gamma_l - i\omega}\left[\frac{2\Gamma_l}{-i\omega} - \frac{(2\Gamma_l - i\omega)}{(\Gamma_r - i\omega)} + 1\right],$$

$$R(\omega) = \frac{\gamma_0 - i\omega}{\Gamma_l - i\omega}\left[1 + \frac{(2\Gamma_l - i\omega)}{(\Gamma_r - i\omega)}\right],$$

$$D = \frac{1}{2}\eta^2\Gamma_l\langle\rho_d^0\rangle_s. \quad (29)$$

From Eq. (28), one can see that the rates $A_\mp$ depend on the tunneling rate $\Gamma_{l(r)}$. For simplicity, below we consider the symmetric couplings of the DQD to the electrodes, i.e., $\Gamma_l = \Gamma_r \equiv \Gamma$.

IV. STEADY-STATE AVERAGE PHONON NUMBER IN THE MECHANICAL RESONATOR

A. Cooling condition

Below we study the cooling limit regarding the steady-state average phonon occupancy in the MR. The equation of motion describing the phonon occupancy distribution can be obtained from master equation (27) of the MR, i.e.,

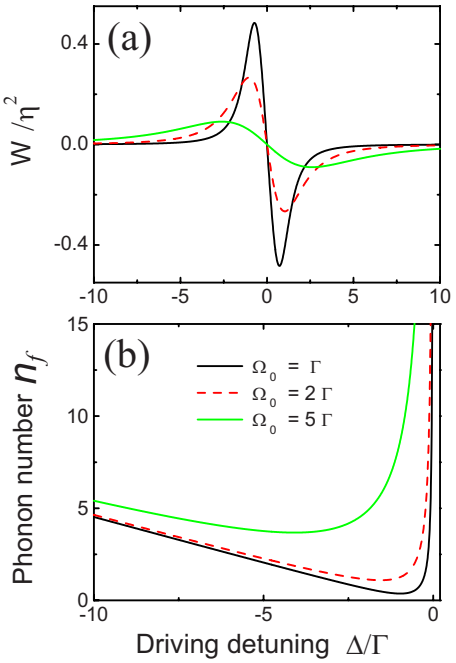

FIG. 3. (Color online) (a) The rate W/η^2 as a function of the driving detuning $\Delta = \omega_d - \omega_0$ for different driving strengths Ω_0. For red-detuning $\Delta < 0$, the rate $W > 0$ and the MR can be cooled. In contrast, the heating process ($W < 0$) dominates the dynamics of the MR for blue-detuning $\Delta > 0$. (b) The steady-state phonon occupancy n_f as a function of the normalized driving detuning Δ/Γ. The parameters are $\Gamma = 1$, $\omega_m = \Gamma$, and $\Omega_0 = \Gamma$ (black curve), $\Omega_0 = 2\Gamma$ (red dashed curve), and $\Omega_0 = 5\Gamma$ (green curve).

$$\frac{dp_n}{dt} = \{\gamma[n(\omega_m)+1] + A_-\}[(n+1)p_{n+1} - np_n] + [\gamma n(\omega_m) + A_+][np_{n-1} - (n+1)p_n], \quad (30)$$

where $p_n = \langle n|\mu|n\rangle$. From Eq. (30), the equation of motion for the average phonon occupancy, $\langle n\rangle = \Sigma_n np_n$, is given by

$$\frac{d\langle n\rangle}{dt} = -(\gamma + W)\langle n\rangle + \gamma n(\omega_m) + A_+, \quad (31)$$

where $W = A_- - A_+$ is the rate of cooling or heating. It is shown that the average phonon occupancy can be increased to infinity and has no stationary solution if $\gamma + W < 0$. In contrast, for $\gamma + W > 0$, one can reach the steady state, i.e., $d\langle n\rangle/dt = 0$. Its solution gives the steady-state average phonon occupancy

$$\langle n\rangle = \frac{\gamma n(\omega_m) + A_+}{\gamma + W}. \quad (32)$$

With expression (32) for the steady-state average phonon occupancy, we can obtain the cooling condition for the single MR.

To achieve cooling, the condition $W > 0$ is required. Otherwise, cooling the MR is unachievable since the heating processes plays a dominant role for $W < 0$. In Fig. 3(a), the rate W is plotted as a function of the driving detuning for different driving strengths. It can be seen that the sign of the rate W exhibits a dependence on the detuning Δ. When the

driving is red-detuned ($\Delta<0$), the rate $W>0$ and the MR is cooled. The steady-state average phonon occupancy is plotted in Fig. 3(b). It clearly reveals the cooling ($\Delta<0$) and heating ($\Delta>0$) regions regarding the driving detuning. Thus, $\Delta=0$ is the crossover from cooling to heating the MR. These results show the importance of the applied driving microwave field in achieving the cooling of the MR. The underlying physical mechanism is explained as follows. When the DQD is excited by a red-detuned microwave field, i.e., $\Delta<0$, the anti-Stokes process ($|1\rangle|n\rangle\rightarrow|2\rangle|n-1\rangle$) is enhanced. Thus, a subsequent decay from the excited state $|2\rangle$ to the ground state $|1\rangle$ reduces the energy of the MR by one quantum and the MR is cooled by repeating this process. On the contrary, when $\Delta>0$, the Stokes process ($|1\rangle|n\rangle\rightarrow|2\rangle|n+1\rangle$) dominates in the system and the MR is heated. Below we focus on the red-detuned region to discuss the cooling of the MR.

At the beginning, if the MR is in its thermal equilibrium state, the initial average phonon occupancy of the MR is given by $n(\omega_m)=n_{\rm th}$. The numerator of Eq. (32) reveals that two parts contribute to the steady-state average phonon occupancy. The first term, $\gamma n(\omega_m)$, results from the thermal bath and the steady-state average phonon occupancy is proportional to the initial thermal occupancy. The second term, A_+, in Eq. (32) originates from the scattering process induced by the driving microwave. In order to achieve an appreciable cooling, i.e., $\langle n\rangle\ll n_{\rm th}$, we need $W\gg\gamma$. In this regime, the steady-state average phonon occupancy is approximately given by

$$\langle n\rangle\approx n_f=\frac{A_+}{W}. \tag{33}$$

In Fig. 4(a), we plot n_f as a function of both the driving detuning and the oscillation frequency of the MR. The region for cooling the MR is enclosed by the contour line $n_f=1$. This region covers a wide area in the ω_m-Δ plane, implying that the cooling of the MR is experimentally accessible. Moreover, the region with steady-state average phonon occupancy much smaller than unity (e.g., $n_f=0.01\ll1$) is explicitly shown in Fig. 4(b). This region corresponds to the cooling of the MR to the ground state, and it can be achieved by both changing the decay rate of the DQD and detuning the transition frequency of the DQD from the microwave frequency.

B. Resolved-sideband cooling

Next, we analytically study the phonon occupancy in the resolved-sideband-cooling region, i.e., $\omega_m\gg\Gamma$. In this regime, the MR's motional sidebands are well resolved since the natural linewidth Γ of the absorption sidebands for different mechanical modes are weakly overlapping. This enables highly targeted cooling with only one mechanical mode.

The previous *semiclassical* analyses[18–20,37,39] show that the resonant system (here, the DQD) *cannot* respond instantaneously to the mechanical motion. Hence, this finite response time induces a phase lag, which produces a force opposing the mechanical motion, leading to a reduction in

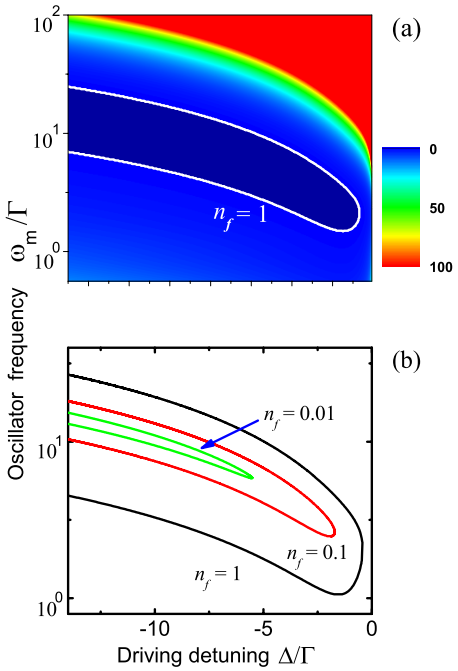

FIG. 4. (Color online) (a) Microwave-induced steady-state average phonon occupancy n_f, shown by colors or shades, as a function of both the normalized driving detuning Δ/Γ and the normalized oscillation frequency ω_m/Γ. (b) Three contour curves of the microwave-induced steady-state average phonon occupancy for $n_f=1$, 0.1, and 0.01. The parameters are $\Gamma=1$ and $\Omega_0=2\Gamma$.

the mechanical motion. Here, in *quantum* theory, we will see how the energy is exchanged between the DQD and the MR during the cooling process (see also Ref. 28). We find below that when the DQD is tuned, via varying the gate voltage, to satisfy the lowest sideband condition, i.e., $\omega_m=\omega_0-\omega_d$, the MR can be cooled sufficiently. In this case, the anti-Stokes process is resonantly enhanced, as discussed in Sec. I.

In the resolved-sideband-cooling regime, the rates $A_\mp$ are approximately given by

$$A_\mp(\omega_m)\approx\eta^2\Gamma\langle\rho_d^0\rangle_s+\frac{1}{2}\frac{\eta^2\Omega_0^2}{(\Delta^2-\omega_m^2+\Omega_0^2)^2+(\omega_m\Gamma)^2}\times\left[\frac{\Gamma}{2}(\Delta^2+\omega_m^2+\Omega_0^2)\langle\rho_d^2+\rho_d^1\rangle_s\mp\Delta\omega_m\Gamma\langle\rho_d^1-\rho_d^2\rangle_s\right], \tag{34}$$

and the rate of cooling or heating is

$$W=A_--A_+=-\frac{\Delta\eta^2\Omega_0^2\omega_m\Gamma}{(\Delta^2-\omega_m^2+\Omega_0^2)^2+(\omega_m\Gamma)^2}\times\frac{4\Delta^2+\Gamma^2}{4\Delta^2+3\Omega_0^2+\Gamma^2}. \tag{35}$$

From Eq. (35), it is obvious that cooling (heating) of the MR occurs for $\Delta<0$ ($\Delta>0$), where $W>0$ ($W<0$). The steady-state average phonon occupancy becomes

$$n_f = \frac{A_+}{W} = -\frac{1}{2} + \frac{1}{-\Delta\omega_m(4\Delta^2+\Gamma^2)} \times \left\{ (\Delta^2 - \omega_m^2 + \Omega_0^2)^2 + \Gamma^2\omega_m^2 + (\Delta^2 + \omega_m^2 + \Omega_0^2)\left(\Delta^2 + \frac{\Omega_0^2}{2} + \frac{\Gamma^2}{4}\right)\right\}. \quad (36)$$

As shown in Eq. (36), the steady-state average phonon occupancy n_f only depends on the normalized detuning Δ/Γ for a fixed MR oscillation frequency. With an optimal detuning value $\Delta=-\omega_m$, the minimum limit of n_f is found to be

$$n_{\min} = \min\{n_f\} = \frac{7}{8}\left(\frac{\Gamma}{\omega_m}\right)^2, \quad (37)$$

which is much smaller than unity in the resolved-sideband-cooling region, i.e., $\omega_m \gg \Gamma$. This result also shows that when the DQD is tuned to the lowest sideband $\Delta=-\omega_m$, the anti-Stokes process ($|1\rangle|n\rangle \rightarrow |2\rangle|n-1\rangle$) is resonantly enhanced. Therefore, an appreciable cooling is achieved. The result obtained here is also consistent with previous theoretical predictions for the resolved-sideband-cooling limit,[25,49] which has been verified by experiments.[29,50] However, an important advantage in the present setup is that both the decay rate and the energy splitting of the DQD can be tuned by varying the gate voltage to reach the resolved-sideband-cooling regime. In this regime, the steady-state average phonon occupancy is much smaller than unity. Thus, the MR can be cooled to its ground state.

Moreover, in Fig. 5 we plot the steady-state average phonon occupancy in the appreciable cooling regime using the exact [Eq. (28)] and the approximate expressions [Eq. (34)] for the rates $A_{\mp}$. In the resolved-sideband regime, the results agree well with each other in these two cases [Fig. 5(a)]. In Fig. 5(b), when the coupled MR-DQD system deviates from the resolved-sideband regime, however, the exact and the approximate expressions can differ significantly from each other. This regime implies the breakdown of the approximation used to obtain Eq. (34). This indicates that one *needs* to use the *exact* expression [Eq. (28)] *to describe the MR cooling in the non-resolved-sideband regime*. Indeed, as shown in Fig. 4, though the condition $\omega_m \gg \Gamma$ is not fulfilled in the non-resolved-sideband regime, the ground-state cooling of the MR is still achievable.

C. Estimates

Finally, let us estimate the steady-state average phonon occupancy of the MR in the resolved-sideband regime using typical experimental parameters.[29,41,42] Here we use $\omega_m = 2\pi \times 100$ MHz, $\Gamma = 2\pi \times 10$ MHz, $\Omega_0 = 2\pi \times 20$ MHz, and $\eta = 0.2$. When the DQD is tuned to the lowest sideband $\Delta=-\omega_m$, the microwave-induced steady-state average phonon occupancy is given by

$$n_f = \frac{7}{8}\left(\frac{\Gamma}{\omega_m}\right)^2 = 0.008\ 75, \quad (38)$$

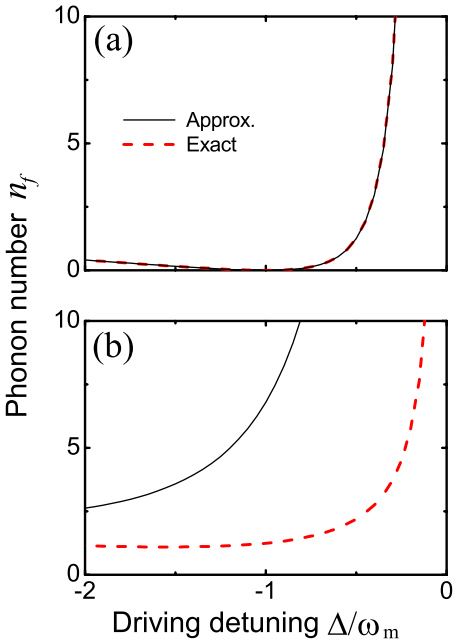

FIG. 5. (Color online) Comparison of the steady-state average phonon occupancy n_f between the exact [Eq. (28)] and approximate expressions [Eq. (34)] as a function of the normalized driving detuning Δ/ω_m. The parameters are $\Gamma=1$ and $\Omega_0=2\Gamma$, as well as (a) $\omega_m/\Gamma=100$ and (b) $\omega_m/\Gamma=1$.

and the cooling rate is

$$W = A_- - A_+ \approx 1.4 \text{ MHz}. \quad (39)$$

Considering a MR with a quality factor $Q=10^5$, the intrinsic dissipation rate of the MR is $\gamma=1$ kHz. Hence, an appreciable cooling effect, i.e., $W \gg \gamma$, can be produced. For a MR precooled by a dilution refrigerator to a temperature T_0 of, e.g., 100 mK, we have $n_{\text{th}} \approx 21$. Thus the steady-state average phonon occupancy part that comes from thermal fluctuations is

$$\frac{\gamma n_{\text{th}}}{W} \approx 0.015. \quad (40)$$

It follows from Eq. (32) that the steady-state average phonon occupancy of the MR is given by

$$\langle n \rangle \approx 0.024 \ll 1, \quad (41)$$

which corresponds to an effective temperature $T_{\text{eff}} \approx 1.3$ mK. This means that the cooling of the MR to the ground state is achievable using the proposed setup with a MR coupled to a DQD.

As derived in Ref. 28, starting from an initial temperature T_0, the final temperature of the cooled MR is bound by

$$T_f = \frac{\omega_m}{\omega_0} T_0, \quad (42)$$

i.e., this expression provides the lower limit of the temperature that can be achieved via the sideband cooling. Experimentally, the energy-level difference $\Delta\varepsilon$ between the ground

state and the first excited state of a single quantum dot can be ~ 250 μeV (see, e.g., Ref. 51), which corresponds to a frequency $\nu \sim 60$ GHz. This level difference can be even larger by decreasing the size of the dot. In our setup, $\Delta\varepsilon$ should be larger than the energy-level difference $\hbar\omega_0$ between the ground state of the left dot and that of the right dot so as to prevent the electron in the right dot from tunneling to the first excited state of the left dot. Here, for example, we can choose $\omega_0 = 2\pi \times 40$ GHz. Other parameters used for calculating $\langle n \rangle$ in Eq. (41) are $\omega_m = 2\pi \times 100$ MHz and $T_0 = 100$ mK. From Eq. (42), we have $T_f = 0.25$ mK. Obviously, the temperature limit is lower than the achieved temperature $T_{\text{eff}} \approx 1.3$ mK in Eq. (41). This implies that the MR can be further cooled using our sideband-cooling proposal.

V. DISCUSSION AND CONCLUSION

To verify if a MR is effectively cooled, one can measure the final average phonon occupancy of the MR, e.g., by detecting the full frequency current noise spectrum of the DQD that is strongly coupled to the MR.[52] For the MR-DQD system studied here, when the MR is effectively cooled, one can tune both the energy-level splitting and the tunneling coupling of the DQD to strongly couple the DQD to the MR. This strong coupling ensures quantum coherent dynamics (i.e., Rabi oscillations) in the coupled MR-DQD system. Actually, these Rabi oscillations have been revealed via the phonon blockade effect on the electron transport through a quantum dot,[53] which is strongly coupled to a single-mode phonon cavity. Moreover, these Rabi oscillations can be reflected as additional resonances in the full frequency current noise spectrum, which are around the central resonance originating from the internal coherent oscillations in the DQD. In contrast, if the MR is not effectively cooled, the quantum behavior is suppressed by the thermal fluctuations and no additional resonances can be observed in the current noise spectrum.[52]

In summary, we have studied the cooling of a MR by electrostatically coupling it to a semiconductor DQD. Here the DQD works as a two-level system and the decay rate of the DQD corresponds to the rate of the electron in the DQD tunneling to the electrode. This tunneling rate and the energy-level splitting of the DQD can be tuned by varying the gate voltage. We show that when the two-level system is driven by a microwave field in red-detuning, the MR can be cooled, in analogy to the laser sideband cooling of atoms or trapped ions in atomic physics. In contrast, when the two-level system is driven in blue-detuning, the MR can be heated. Also, we obtain analytical results for the resolved-sideband cooling of the MR. Moreover, our results show that the ground-state cooling of the MR can be achieved both by detuning the transition frequency of the DQD from the microwave frequency and by changing the decay rate of the DQD. Importantly, this frequency detuning and the decay rate of the DQD are tunable by varying the gate voltages of the DQD. Thus, the coupled MR-DQD system provides an experimentally implementable setup for ground-state cooling of MRs.

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APPENDIX A: MASTER EQUATION FOR THE DENSITY MATRIX PROJECTED ON A SUBSPACE OF THE LIOUVILLIAN $\mathcal{L}_0$

In this appendix, we derive Eq. (26) by projecting master equation (17) on the subspace, with zero eigenvalue $\lambda_0 = 0$, of the Liouvillian $\mathcal{L}_0$ for the decoupled resonator and DQD system. The projection is defined as

$$\mathcal{P}\rho = \rho_d^s \otimes \mathrm{Tr}_d\{\rho\} = \rho_d^s \otimes \mu, \quad \mathcal{Q} = 1 - \mathcal{P}, \tag{A1}$$

where $\mathcal{P}$ is the projection operator. The definition of $\mathcal{P}$ implies that $\mathcal{L}_0\mathcal{P} = \mathcal{P}\mathcal{L}_0 = 0$, which leads to

$$\mathcal{P}\mathcal{L}_0\mathcal{P} = \mathcal{Q}\mathcal{L}_0\mathcal{P} = \mathcal{P}\mathcal{L}_0\mathcal{Q} = 0, \quad \mathcal{Q}\mathcal{L}_0\mathcal{Q} = \mathcal{L}_0. \tag{A2}$$

With these relations, the projection of master equation (17) gives

$$\mathcal{P}\dot{\rho} = \mathcal{P}\mathcal{L}_2\mathcal{P}\rho + \mathcal{P}(\mathcal{L}_1 + \mathcal{L}_2)\mathcal{Q}\rho, \tag{A3}$$

$$\mathcal{Q}\dot{\rho} = [\mathcal{L}_0 + \mathcal{Q}(\mathcal{L}_1 + \mathcal{L}_2)]\mathcal{Q}\rho + \mathcal{Q}(\mathcal{L}_1 + \mathcal{L}_2)\mathcal{P}\rho. \tag{A4}$$

Here we have used the relation $\mathcal{P}\mathcal{L}_1\mathcal{P} = 0$, due to the fact that tracing over the interaction between the DQD and the MR equals zero. Next, we define

$$v(t) \equiv \mathcal{P}\rho(t), \quad w(t) \equiv \mathcal{Q}\rho(t). \tag{A5}$$

Applying the Laplace transform

$$\tilde{b}(s) = \int_0^\infty b(t)e^{-st}dt \tag{A6}$$

on Eq. (A4), one has

$$s\tilde{v}(s) - v(0) = \mathcal{P}\mathcal{L}_2\tilde{v}(s) + \mathcal{P}(\mathcal{L}_1 + \mathcal{L}_2)\tilde{w}(s),$$

$$s\tilde{w}(s) - w(0) = [\mathcal{L}_0 + \mathcal{Q}(\mathcal{L}_1 + \mathcal{L}_2)]\tilde{w}(s) + \mathcal{Q}(\mathcal{L}_1 + \mathcal{L}_2)\tilde{v}(s). \tag{A7}$$

Then we introduce a small parameter ζ to characterize the order of the Liouvillians,[54] i.e.,

$$\mathcal{L}_0(t) \to \mathcal{L}_0(t), \quad \mathcal{L}_1 \to \zeta\mathcal{L}_1(t), \quad \mathcal{L}_2 \to \zeta^2\mathcal{L}_2(t). \tag{A8}$$

Substituting Eq. (A8) into Eq. (A7) and including terms up to second order, one has

$$s\tilde{v}(s) - [v(0) + \mathcal{P}(\zeta\mathcal{L}_1 + \zeta^2\mathcal{L}_2)(s - \mathcal{L}_0)^{-1}w(0)] = \zeta^2\mathcal{P}\mathcal{L}_2\tilde{v}(s) + \zeta^2\mathcal{P}\mathcal{L}_1(s - \mathcal{L}_0)^{-1}\mathcal{Q}\mathcal{L}_1\tilde{v}(s). \tag{A9}$$

Finally, neglecting the correction due to the initial condition

and performing the inverse Laplace transform, one obtains

$$\mathcal{P}\dot{\rho} = \mathcal{P}\mathcal{L}_2\mathcal{P}\rho + \mathcal{P}\mathcal{L}_1\mathcal{Q}(-\mathcal{L}_0)^{-1}\mathcal{Q}\mathcal{L}_1\mathcal{P}\rho, \tag{A10}$$

which is just Eq. (26) in Sec. III.

APPENDIX B: MASTER EQUATION FOR THE REDUCED DENSITY MATRIX OF THE MECHANICAL RESONATOR

Below we derive the master equation for the reduced density matrix μ of the MR. Following the procedures in Ref. 48, we trace over the DQD degrees of freedom in Eq. (A10). The first term on the right-hand side of Eq. (A10) gives

$$\begin{aligned}
&\mathrm{Tr}_d(\mathcal{P}\mathcal{L}_2\mathcal{P}) \\
&= -i\frac{\eta^2\Omega_0}{2}\langle\sigma_x\rangle_s[b^\dagger b,\mu] \\
&\quad + \frac{1}{2}\{\gamma[n(\omega_m)+1] + \eta^2\Gamma_l\langle\rho_d^0\rangle_s\} \\
&\quad \times[2b\mu b^\dagger - (b^\dagger b\mu + \mu b^\dagger b)] \\
&\quad + \frac{1}{2}[\gamma n(\omega_m) + \eta^2\Gamma_l\langle\rho_d^0\rangle_s][2b^\dagger\mu b - (bb^\dagger\mu + \mu bb^\dagger)].
\end{aligned} \tag{B1}$$

The second term on the right-hand side of Eq. (A10) gives

$$\begin{aligned}
&\mathrm{Tr}_d\{\mathcal{P}\mathcal{L}_1(-\mathcal{L}_0)^{-1}\mathcal{L}_1\mathcal{P}\rho\} \\
&= \int_0^{+\infty} dt\ \mathrm{Tr}_d\{\mathcal{P}\mathcal{L}_1 e^{-\mathcal{L}_0 t}\mathcal{L}_1\mathcal{P}\rho\} \\
&= -i\,\mathrm{Im}[S(\omega_m) + S(-\omega_m)][b^\dagger b,\mu] \\
&\quad + \mathrm{Re}[S(\omega_m)][2b\mu b^\dagger - (b^\dagger b\mu + \mu b^\dagger b)] \\
&\quad + \mathrm{Re}[S(-\omega_m)][2b^\dagger\mu b - (bb^\dagger\mu + \mu bb^\dagger)],
\end{aligned} \tag{B2}$$

where

$$S(\omega) = \eta^2\frac{\Omega_0^2}{4}\int_0^{+\infty} dt e^{i\omega t}\langle\sigma_y(t)\sigma_y(0)\rangle. \tag{B3}$$

Substituting Eqs. (B1), (B2), and (21) into Eq. (A10), we obtain the master equation for the MR,

$$\begin{aligned}
\dot{\mu} &= -i[\omega_m + \delta_m, b^\dagger b] \\
&\quad + \frac{1}{2}\{\gamma[n(\omega_m)+1] + A_-(\omega_m)\}[2b\mu b^\dagger - (b^\dagger b\mu + \mu b^\dagger b)] \\
&\quad + \frac{1}{2}[\gamma n(\omega_m) + A_+(\omega_m)][2b^\dagger\mu b - (bb^\dagger\mu + \mu bb^\dagger)].
\end{aligned} \tag{B4}$$

This is just Eq. (27) in Sec. III.

In Eq. (B4), we have introduced

$$\delta_m = \eta^2\frac{\Omega_0}{2}\langle\sigma_x\rangle_s + \mathrm{Im}[S(\omega_m) + S(-\omega_m)] \tag{B5}$$

and the rates $A_\mp(\omega_m)$,

$$A_\mp(\omega_m) = 2\,\mathrm{Re}[S(\pm\omega_m) + D], \tag{B6}$$

with

$$D = \frac{1}{2}\eta^2\Gamma_l\langle\rho_d^0\rangle_s, \tag{B7}$$

where $\langle\rho_d^0\rangle_s$ is the probability of an empty DQD at the steady state.

To determine the rates $A_\mp$, we need to calculate the correlation function $S(\omega)$ of Eq. (B3) using the equation of motion for the DQD:

$$\dot{\rho}_d = -i\left[-\frac{\Delta}{2}\sigma_z + \frac{\Omega_0}{2}\sigma_x, \rho_d\right] + \Gamma_l\mathcal{D}[a_1^\dagger]\rho_d + \Gamma_r\mathcal{D}[a_2]\rho_d. \tag{B8}$$

From Eq. (B8), one can obtain the following equations of motion:

$$\begin{aligned}
\dot{\rho}_d^1 &= \Gamma_l - \Gamma_l\rho_d^1 - \Gamma_l\rho_d^2 - \frac{\Omega_0}{2}\langle\sigma_y\rangle, \\
\dot{\rho}_d^2 &= -\Gamma_r\rho_d^2 + \frac{\Omega_0}{2}\langle\sigma_y\rangle, \\
\langle\dot{\sigma}_x\rangle &= \Delta\langle\sigma_y\rangle - \gamma_0\langle\sigma_x\rangle, \\
\langle\dot{\sigma}_y\rangle &= -\Delta\langle\sigma_x\rangle - \Omega_0(\rho_d^2 - \rho_d^1) - \gamma_0\langle\sigma_y\rangle,
\end{aligned} \tag{B9}$$

where $\gamma_0 = \Gamma_r/2$. With the normalization condition $\rho_d^0 + \rho_d^1 + \rho_d^2 = 1$, the steady-state solution of Eq. (B8) is given by

$$\begin{aligned}
&\langle\rho_d^0\rangle_s = \frac{\Omega_0^2\Gamma_r}{M}, \quad \langle\rho_d^1\rangle_s = \frac{4\Gamma_l\Delta^2 + \Gamma_l\Omega_0^2 + \Gamma_l\Gamma_r^2}{M}, \\
&\langle\rho_d^2\rangle_s = \frac{\Gamma_l\Omega_0^2}{M}, \quad \langle\sigma_x\rangle_s = \frac{4\Gamma_l\Delta\Omega_0}{M}, \quad \langle\sigma_y\rangle_s = \frac{2\Gamma_l\Gamma_r\Omega_0}{M},
\end{aligned} \tag{B10}$$

where

$$M = 4\Gamma_l\Delta^2 + \Omega_0^2(2\Gamma_l + \Gamma_r) + \Gamma_l\Gamma_r^2. \tag{B11}$$

From Eq. (B9), one can obtain

$$\begin{aligned}
\langle\tilde{\sigma}_y(s)\rangle &= \left\{\Delta^2 + (s+\gamma)^2 + \frac{\Omega_0^2}{2}(s+\gamma)\frac{2s + 2\Gamma_l + \Gamma_r}{(s+\Gamma_l)(s+\Gamma_r)}\right\}^{-1} \\
&\quad \times\left\{\Omega_0\frac{\Gamma_l(s+\gamma)}{s(s+\Gamma_l)} + \Omega_0\frac{(s+\gamma)}{(s+\Gamma_l)}\rho_d^1(0)\right. \\
&\quad - \Omega_0\frac{(s+\gamma)(s+2\Gamma_l)}{(s+\Gamma_l)(s+\Gamma_r)}\rho_d^2(0) \\
&\quad \left. + (s+\gamma)\langle\sigma_y\rangle(0) - \Delta\langle\sigma_x\rangle(0)\right\},
\end{aligned} \tag{B12}$$

which is the Laplace transform of $\langle\sigma_y(t)\rangle$. Then, using the quantum regression theorem[47] and performing the inverse Laplace transform, one can calculate the correlation function in Eq. (B3). Subsequently, with Eq. (B6), the rates $A_\mp(\omega_m)$ are obtained as in Eq. (28).

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